ON THE ISSUE OF LEARNING WEIGHTS FROM OBSERVATIONS FOR FUZZY SIGNATURES

B. Sumudu U. Mendis, Dept. of Comp. Science, The Australian National University, Canberra, ACT 0200, Australia, sumudu.mendis@anu.edu.au
Tamás D. Gedeon, Dept. of Computer Science, The Australian National University, Canberra, ACT 0200, Australia, tom.gedeon@anu.edu.au
László T. Kóczy, Dept. of Telecommunication and Media Informatics, Budapest University of Technology and Economics Budapest, & Institute of Information Technology and Electrical Engineering, Széchenyi István University, Győr, Hungary, koczy@tmit.bme.hu

ABSTRACT

We investigate the issue of obtaining weights, which are associated with aggregation in fuzzy signatures, from real world data. Our approach will provide a way to extract the relevance of lower levels to the higher levels of the hierarchical fuzzy signature structure. We also handle the non-differentiability of *max-min* aggregation functions for gradient based learning. A mathematically proved method, which is found in the literature to approximate the derivatives of *max–min* functions, has been used.

Keywords: Vector valued fuzzy sets, Fuzzy signatures, Weighted aggregation.

1. INTRODUCTION

In Vámos [1] and Kóczy [3] the vector valued fuzzy sets concept [4] has been further generalized to introduce the fuzzy signature concept. Fuzzy signatures model complex structured problems with the help of hierarchically structured vector valued fuzzy sets and a set of aggregation functions, which are not necessarily homogeneous.

In [6] we further enhanced the inference in fuzzy signatures, by introducing the weighted aggregation method. The concept behind the weighted aggregation method is that it introduces additional expert knowledge to the fuzzy signature structure. The weights in each branch of a fuzzy signature imply the relevance of that branch to its higher level branches of the fuzzy signature. Thus, fuzzy signatures are sophisticated problem solvers in vague environments similar to some human abilities.

In this research we focus on finding a practical method to extract the weights from real world data and to observe the improvement of the accuracy of the final results of the fuzzy signatures. Section 2 shows the methodology of using the gradient descent learning method for learning weights for the fuzzy signature structure from observations. Section 3 shows the issue of non differentiability of maximum and minimum aggregation functions and a way of overcoming the problem. Finally, Section 4 shows how to obtain derivatives for gradient descent learning, using an example problem.

2. WEIGHTS LEARNING FROM OBSERVATIONS

In this section, we first briefly describe the weighted aggregation method for fuzzy signature inference process. Next, we describe an algorithm for weights learning using gradient descent. Fuzzy signatures are vector valued fuzzy sets, where each vector component can be a further vector valued fuzzy set [3]. A fuzzy signature, s can be defined as,

$$s : X \to [a_i]_{i=1}^{k}, \quad where \quad a_i = \begin{cases} [0,1]; & if \quad leafe \\ [a_{ij}]_{j=1}^{k_i}; & if \quad branch \end{cases}$$

Fig.1 shows an example of a fuzzy signature structure with two arbitrary levels g and $(g+1)$. Now, aggregation of the branch a_0 in level 0 of the fuzzy signature structure can be written as,

$a_0 = @_0\{a_i\}$, where $@_0$ is an arbitrary aggregation function, *i=1…l*, and $a_i \in [0,1]$. Also, aggregation of an arbitrary branch $a_{p...i}$ in level *g* (fig.1) can be written as, $a_{p...i} = @_{p..i}\{a_{p...ij}\}$, where $@_{p..i}$ is an arbitrary aggregation function, *j=1…n*, and $a_{p...ij} \in [0,1]$. These aggregation functions, which are used for inference in fuzzy signatures, can be simple aggregations like minimum (min), average (avg), maximum_average (max-avg), or maximum (max), which were used in [6], or they can be complex aggregation functions as proposed in [3].

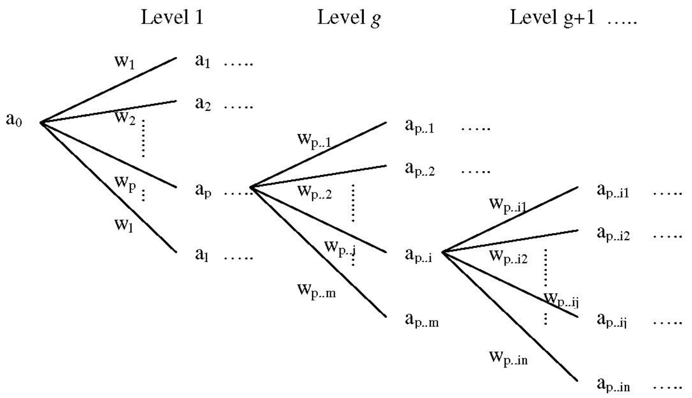

Fig. 1. Fuzzy Signature Structure with two arbitrary levels g and (g+1)

The weighted aggregation concept was proposed in [6] to provide additional expert knowledge to the fuzzy signature structure by introducing the weighted relevance of each branch to its higher branches of the fuzzy signature structure. In other words, weighted relevance reflects the fact that some branches may contribute more to the final result than the other branches in the same level. Therefore, the weighted relevance will give an additional ability to fuzzy signatures for decision making in situations where input data are vague and complex. This is equivalent to the situations where in real world physicians need to make decisions depending on limited available diagnosis data of patients in emergency situations.

Weighted aggregation of an arbitrary branch $a_{p..i}$ in a fuzzy signature (fig.1) can be defined as, $a_{p..i} = @_{p..i}\{w_{p...ij} \bullet a_{p..ij}\}$, (1)

where $@_{p..i}$ is an arbitrary aggregation function, *i=1…n*, and $a_{p..ij} \in [0,1]$. The following properties hold for arbitrary weighted relevance w_p,

I. $w_p \in [0,1]$ II. $\sum_{p=1}^{l} w_p$ is not necessarily equal to 1.

Now, we shall focus on the issue of learning weights from real data. Let us assume that we have *t* number of records (observations) in a training data set. Also, *t* is greater than the number of weight coefficients needed to be learnt for the fuzzy signature structure. Further, let us denote d_k as the desired value for the training data record $k(\le t)$ and e_k be the squared error between the desired value and the actual value of the same record. Next, we can write the squared error of the fuzzy signature structure in fig.1 for training data record *k* as follows, $e_k = \frac{1}{2}\left(a_0^k - d_k\right)^2$, where a_0^k is the final atomic result of the fuzzy signature *s* (fig.1) for training data record *k*. Error function e_k can be further expanded as follows, $e_k = \frac{1}{2}\{@_0[w_i \bullet a_i] - d_k\}^2$, where $@_0$ is an arbitrary aggregation function, *i=1…l*, and $w_i \in [0,1]$. Further, above error function e_k can be generalized as follows,

$e_k = \frac{1}{2}\{@_0[w_i(@_i[w_{ij}(.........@_{ij...p}[w_{ij...pq} \bullet a_{ij...pq}])])] - d_k\}^2$, where $@_0$, $@_i$ and $@_{ij..p}$ are arbitrary aggregation functions, *i=1…l* , *j=1…b, p=1…m, q=1…n* , and $w_i, w_{ij}, w_{ij...pq} \in [0,1]$. To avoid the constraints on the weighted relevance factor w_i, we replaced it by following sigmoid function, $w_i = \dfrac{1}{1+e^{-\lambda_i}}$, (2)

where $\lambda_i \in \Re$. After the above transformation it becomes clear that for any values of the parameter λ_i the weighted relevance $w_i \in [0,1]$ and $\sum w_i$ is not necessarily equal to 1. Therefore the constrained optimization problem has been transformed to an unconstrained optimization problem. Now error function e_k can be rewritten as follows,

$$e_k = \frac{1}{2}\left\{ @_0\left[\frac{1}{1+e^{\lambda_i}}\left(@_i\left[\frac{1}{1+e^{\lambda_{ji}}}\left(..........@_{ij...p}\left[\frac{1}{1+e^{\lambda_{ij...pq}}}\bullet a_{ij...pq}\right]\right)\right]\right)\right] - d_k\right\}^2 \tag{3}$$

Next, the steepest gradient descent method has been used to minimize (as in [5]) the error e_k. Using the steepest gradient descent method we obtain the following equation to minimize the error e_k according to the λ parameters in the equation (3). For arbitrary $\lambda_{ij...pq}$ at any level in fig.1, the next update $\lambda^{next}_{ij...pq}$ can be written as, $\lambda^{next}_{ij...pq} = \lambda_{ij...pq} - \beta\left(\dfrac{\partial e_k}{\partial \lambda_{ij...pq}}\right)$, where β is the learning rate and $\left(\dfrac{\partial e_k}{\partial \lambda_{ij...pq}}\right)$ is given by the following equation,

$$\left(\frac{\partial e_k}{\partial \lambda_{ij...pq}}\right) = \frac{\partial\left(\frac{1}{2}\left\{ @_0\left[\frac{1}{1+e^{\lambda_i}}\left(@_i\left[\frac{1}{1+e^{\lambda_{ji}}}\left(..........@_{ij...p}\left[\frac{1}{1+e^{\lambda_{ij...pq}}}\bullet a_{ij...pq}\right]\right)\right]\right)\right] - d_k\right\}^2\right)}{\partial \lambda_{ij...pq}} \tag{4}$$

In this way we can obtain next updates for each λ in the fuzzy signature structure. These updates can be used to find the new weights for the fuzzy signature using equation (2). As above equation (4) contains *max* and/or *min* functions we may face the problem of non-differentiability of those functions. In the next section, we discuss how to approximate derivatives for the above equation (4).

3. DERIVATIVES OF *MAX-MIN* FUNCTIONS FOR FUZZY SIGNATURES

As *max–min* functions are not strictly differentiable, a method has been proposed [2] to approximate the side-derivatives and pseudo derivatives of *max-min* functions for gradient descent techniques. We used these theorems to obtain the derivative of equation (4) when *max* and/or *min* functions are present in equation (4). First we repeat the theorems we used and then we discuss the differentiation of an arbitrary branch of the fuzzy signature in fig.1. The following theorems (i), (ii), and (iii) have been proved in [2].

Theorem (i) : Let *F*(*x*) and *G*(*x*) be functions that are side-differentiable in the first order and quasi-differentiable in the second order. Suppose *F*(*x*) and *G*(*x*) coincide over the interval $l_b \le x \le u_b$, such that $\forall x \in [l_b, u_b]$, $\mathrm{F(x)} = \mathrm{G(x)}$, $\dfrac{\partial F(x)}{\partial x^+} = \dfrac{\partial G(x)}{\partial x^+}$ and $\dfrac{\partial F(x)}{\partial x^-} = \dfrac{\partial G(x)}{\partial x^-}$. Suppose further that *F*(*x*) is periodic over the interval $l'_b \le x \le u'_b$, where $l'_b < l_b$ and $u'_b > u_b$. Then $\forall x \in [l_b, u_b]$, $\dfrac{dG(x)}{dx^{\pm}} = \dfrac{1}{2}\left(\dfrac{\partial G(x)}{\partial x^+} + \dfrac{\partial G(x)}{\partial x^-}\right)$

Theorem (ii): Given $\overline{G}(x) = \min_i^n (f(x))$, such that $f_k(x)$ is continuous and side differentiable $\forall k = 1...n$, then $\overline{G}(x)$ is also continuous, and the following side derivatives exists:

a) $\dfrac{\partial \overline{G}(x)}{\partial x^+} = \min\limits_{i \in \Theta(x)} \left(\dfrac{\partial f(x)}{\partial x^+} \right)$ b) $\dfrac{\partial \overline{G}(x)}{\partial x^-} = \max\limits_{i \in \Theta(x)} \left(\dfrac{\partial f(x)}{\partial x^-} \right)$

where $\overline{\Theta}(x) = \{ i | i \in \{1...n\} \;\; and \;\; f_i(x) = \overline{G}(x) \}$.

Theorem (iii): Given $\overline{G}(x) = \max_i^n (f(x))$, such that $f_k(x)$ is continuous and side differentiable $\forall k = 1...n$, then $\overline{G}(x)$ is also continuous, and the following side derivatives exists:

a) $\dfrac{\partial \overline{G}(x)}{\partial x^+} = \max\limits_{i \in \Theta(x)} \left(\dfrac{\partial f(x)}{\partial x^+} \right)$ b) $\dfrac{\partial \overline{G}(x)}{\partial x^-} = \min\limits_{i \in \Theta(x)} \left(\dfrac{\partial f(x)}{\partial x^-} \right)$

where $\overline{\Theta}(x) = \{ i | i \in \{1...n\} \;\; and \;\; f_i(x) = \overline{G}(x) \}$.

Now we can write the deravative of e_k with respect to $\lambda_{p...ij}$ (Fig. 1) as, $\dfrac{\partial e_k}{\partial \lambda_{p...ij}} = \left(a_0^k - d_k \right) \dfrac{\partial a_0^k}{\partial \lambda_{p...ij}}$. Using theorem (i) we can write the deravative of a_0 with respect to $\lambda_{p...ij}$ as follows,

$$\frac{\partial a_0}{\partial \lambda^{\pm}_{p...ij}} = \frac{1}{2} \left\{ \frac{\partial a_0}{\partial \lambda^{+}_{p...ij}} + \frac{\partial a_0}{\partial \lambda^{-}_{p...ij}} \right\} \tag{5}$$

Now let us assume that following aggregation functions $\{\min, avg, \max - avg, \max\}$ are used for the aggregation of fuzzy signature in fig.1. Also, recall the equation (1), which represents the agrregated result of an arbitary branch $a_{p...i} = @_{p...i} \{ w_{p...ij} \bullet a_{p...ij} \}$, where $@_{p...i} \in \{\min, avg, \max avg, \max\}$. Now we get four different derivatives for equation (1) depending on the selection of the $@_{p...i}$. For all cases, Let *g* be an arbitrary level in the fuzzy signature a_0 (fig.1) and *(g+1)* be next consecutive level of the same fuzzy signature. Using theorem (ii) and (iii) the side derivatives of equation (1) can be written as,

<u>Case I (</u> $@_{p...i} = \min$):

$$\frac{\partial a_{p...i}}{\partial \lambda^{+}_{p...ij}} = \min_{k \in \Omega_{p...i}} \left\{ \frac{\partial \left(w_{p...jk} \bullet a_{p...ik} \right)}{\partial \lambda^{+}_{p...ij}} \right\} = \begin{cases} 0 & ;if \;\; \Omega_{p...i} \neq \{j\} \\ \dfrac{\partial \left(w_{p...ij} \bullet a_{p...ij} \right)}{\partial \lambda^{+}_{p..ij}} & ;if \;\; \Omega_{p...i} = \{j\} \end{cases} \tag{I1}$$

$$\frac{\partial a_{p...i}}{\partial \lambda^{-}_{p...ij}} = \max_{k \in \Omega_{p...i}} \left\{ \frac{\partial \left(w_{p...jk} \bullet a_{p...ik} \right)}{\partial \lambda^{-}_{p...ij}} \right\} = \begin{cases} 0 & ;if \;\; j \notin \Omega_{p...i} \\ \dfrac{\partial \left(w_{p...ij} \bullet a_{p...ij} \right)}{\partial \lambda^{-}_{p...ij}} & ;if \;\; j \in \Omega_{p...i} \end{cases} \tag{I2}$$

,where $\Omega_{p...i} = \{ k | k \in [1, n] \;\; and \;\; a_{p...i} = w_{p..qi} \bullet a_{p..ik} \}, i \in [1, m]$, and $p \in [1, l]$.

<u>Case II (</u> $@_{p...i} = \max$):

$$\frac{\partial a_{p...i}}{\partial \lambda^{+}_{p...ij}} = \max_{k \in \Omega_{p...i}} \left\{ \frac{\partial \left(w_{p...jk} \bullet a_{p...ik} \right)}{\partial \lambda^{+}_{p...ij}} \right\} = \begin{cases} 0 & ;if \;\; j \notin \Omega_{p...i} \\ \dfrac{\partial \left(w_{p...ij} \bullet a_{p...ij} \right)}{\partial \lambda^{+}_{p..ij}} & ;if \;\; j \in \Omega_{p...i} \end{cases} \tag{II1}$$

$$\frac{\partial a_{p...i}}{\partial \lambda^{-}_{p...ij}} = \min_{k \in \Omega_{p...i}} \left\{ \frac{\partial \left(w_{p...jk} \bullet a_{p...ik} \right)}{\partial \lambda^{-}_{p...ij}} \right\} = \begin{cases} 0 & ;if \;\; \Omega_{p...i} \neq \{j\} \\ \dfrac{\partial \left(w_{p...ij} \bullet a_{p...ij} \right)}{\partial \lambda^{-}_{p...ij}} & ;if \;\; \Omega_{p...i} = \{j\} \end{cases} \tag{II2}$$

, where $\Omega_{p...i} = \{ k | k \in [1, n] \;\; and \;\; a_{p...i} = w_{p..qi} \bullet a_{p..ik} \}$, $i \in [1, m]$, and $p \in [1, l]$.

Case III ($@_{p...i} = avg$):

$$\frac{\partial a_{p...i}}{\partial \lambda^{+}_{p...ij}} = \sum_{k=1}^{n}\left\{\frac{\partial\left(w_{p..ik} \bullet a_{p..ik}\right)}{\partial \lambda^{+}_{p..ij}}\right\} = \frac{1}{n}\left\{\frac{\partial\left(w_{p..ij} \bullet a_{p..ij}\right)}{\partial \lambda^{+}_{p..ij}}\right\} \tag{III1}$$

$$\frac{\partial a_{p...i}}{\partial \lambda^{-}_{p...ij}} = \sum_{k=1}^{n}\left\{\frac{\partial\left(w_{p..ik} \bullet a_{p..ik}\right)}{\partial \lambda^{-}_{p..ij}}\right\} = \frac{1}{n}\left\{\frac{\partial\left(w_{p..ij} \bullet a_{p..ij}\right)}{\partial \lambda^{-}_{p..ij}}\right\} \tag{III2}$$

Case IV ($@_{p...i} = \max - avg$): In this case equation (1) can be modified as follows,

$$a_{p...i} = \frac{1}{2}\left[\max\left\{a_{p...ik} \times w_{p...ik}\right\} + avg\left\{a_{p...ik} \times w_{p...ik}\right\}\right]$$

$$\frac{\partial a_{p...i}}{\partial \lambda^{+}_{p...ij}} = \frac{1}{2}\left[\max_{k \in \Omega_{p...i}}\left\{\frac{\partial\left(w_{p...ik} \bullet a_{p.ik}\right)}{\partial \lambda^{+}_{p..ij}}\right\} + \frac{1}{n}\left\{\frac{\partial\left(w_{p...ik} \bullet a_{p.ik}\right)}{\partial \lambda^{+}_{p...ij}}\right\}\right] \tag{IV1}$$

$$\frac{\partial a_{p...i}}{\partial \lambda^{-}_{p...ij}} = \frac{1}{2}\left[\min_{k \in \Omega_{p...i}}\left\{\frac{\partial\left(w_{p...ik} \bullet a_{p.ik}\right)}{\partial \lambda^{-}_{p..ij}}\right\} + \frac{1}{n}\left\{\frac{\partial\left(w_{p...ik} \bullet a_{p.ik}\right)}{\partial \lambda^{-}_{p...ij}}\right\}\right] \tag{IV2}$$

, where $\Omega_{p...i} = \left\{k \middle| k \in [1,n] \;\; and \;\; a_{p...i} = w_{p..qi} \bullet a_{p.ik}\right\}$, $i \in [1,m]$, and $p \in [1,l]$. Equation (IV1) and (IV2) can be futher expanded as follows,

$$\frac{\partial a_{p..q}}{\partial \lambda^{+}_{p..qr}} = \begin{cases} \frac{1}{n}\left\{\frac{\partial\left(a_{p..qr} \times w_{p..qr}\right)}{\partial \lambda^{+}_{p..qr}}\right\} & ;if \;\; r \notin \Omega_{p..q} \\ \frac{\partial\left(a_{p..qr} \times w_{p..qr}\right)}{\partial \lambda^{+}_{p..qr}} + \frac{1}{n}\left\{\frac{\partial\left(a_{p..qr} \times w_{p..qr}\right)}{\partial \lambda^{+}_{p..qr}}\right\} & ;if \;\; r \in \Omega_{p..q} \end{cases} \tag{IV3}$$

$$\frac{\partial a_{p..q}}{\partial \lambda^{-}_{p..qr}} = \begin{cases} \frac{1}{n}\left\{\frac{\partial\left(a_{p..qr} \times w_{p..qr}\right)}{\partial \lambda^{-}_{p..qr}}\right\} & ;if \;\; \Omega_{p..q} \neq \{r\} \\ \frac{\partial\left(a_{p..qr} \times w_{p..qr}\right)}{\partial \lambda^{-}_{p..qr}} + \frac{1}{n}\left\{\frac{\partial\left(a_{p..qr} \times w_{p..qr}\right)}{\partial \lambda^{-}_{p..qr}}\right\} & ;if \;\; \Omega_{p..q} = \{r\} \end{cases} \tag{IV4}$$

4. EXAMPLE PROBLEM : SALARY SELECTION FUZZY SIGNATURE

In this section we describe how to calculate the derivatives for gradient descent learning, discussed in section 2, using the method shown in section 3. The salary selection fuzzy signature in [6] has been selected as the example real world problem.

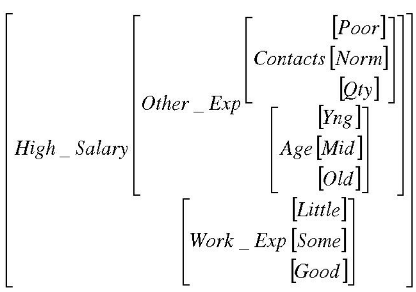

Fig.2.a. High Salary Selection Fuzzy Signature

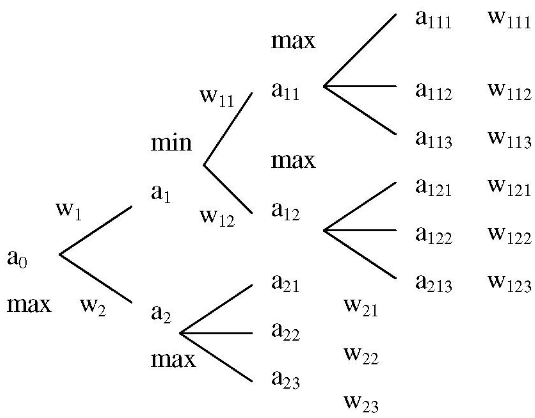

Fig.2.b. High Salary Selection Fuzzy Signature with input values

Fig.2. High Salary Selection Fuzzy Signature

The salary selection fuzzy signature (fig.2.a.) describes the selection of high salary of a person given by work experience, contacts, and age.

Using the gradient descent method the next update of λ at node a_{12} (fig.2.b.) can be written as, $\lambda_{12}^{t+1} = \lambda_{12}^{t} - \beta \frac{\partial e_k}{\partial \lambda_{12}}$. Also, the rate of change of square error can be written as, $\frac{\partial e_k}{\partial \lambda_{12}} = \frac{\partial a_0^k}{\partial \lambda_{12}}\left(a_0^k - d^k\right)$. Further according to (3), $\frac{\partial a_0^k}{\partial \lambda_{12}^{\pm}} = \frac{1}{2}\left\{\frac{\partial a_0^k}{\partial \lambda_{12}^{+}} + \frac{\partial a_0^k}{\partial \lambda_{12}^{-}}\right\}$ and using (II1) and (II2) we can write $\frac{\partial a_0^k}{\partial \lambda_{12}^{+}} = \max_{i \in \Omega_1}\left\{\frac{\partial\left(a_i^k \times w_i\right)}{\partial \lambda_{12}^{+}}\right\}$ and $\frac{\partial a_0^k}{\partial \lambda_{12}^{-}} = \min_{i \in \Omega_1}\left\{\frac{\partial\left(a_i^k \times w_i\right)}{\partial \lambda_{12}^{-}}\right\}$, where $\Omega_1 = \left\{i \middle| i \in [1,2] \; and \; a_0^k = a_i^k \times w_i\right\}$. Now as in (II1) $\frac{\partial a_0^k}{\partial \lambda_{12}^{+}}$ can be expanded as,

$$\frac{\partial a_0^k}{\partial \lambda_{12}^{+}} = \begin{cases} 0 & ;if \;\; 1 \notin \Omega_1 \\ \frac{\partial\left(a_1 \times w_1\right)}{\partial \lambda_{122}^{+}} & ;if \;\; 1 \in \Omega_1 \end{cases}$$

Let us assume that, $1 \in \Omega_1$ then we can write $\frac{\partial a_0^k}{\partial \lambda_{12}^{+}} = w_1 \bullet \frac{\partial a_1^k}{\partial \lambda_{12}^{+}}$ and $\frac{\partial a_1}{\partial \lambda_{12}^{+}} = \min_{k \in \Omega_2}\left\{\frac{\partial\left(a_{1k}^k \bullet w_{1k}\right)}{\partial \lambda_{12}^{+}}\right\}$, where $\Omega_2 = \left\{k \middle| k \in [1,2] \; and \; a_1^k = a_{1k}^k \bullet w_{12}\right\}$. Now using (I1) $\frac{\partial a_1^k}{\partial \lambda_{12}^{+}}$ can be written as,

$$\frac{\partial a_1^k}{\partial \lambda_{12}^{+}} = \begin{cases} 0 & ;if \;\; \Omega_2 \neq \{2\} \\ \frac{\partial\left(a_{12}^k \bullet w_{12}\right)}{\partial \lambda_{12}^{+}} & ;if \;\; \Omega_2 = \{2\} \end{cases}$$

Assume that, $\{2\} = \Omega_2$ then we can write: $\frac{\partial a_1^k}{\partial \lambda_{12}^{+}} = a_{12}^k \bullet \frac{\partial w_{12}}{\partial \lambda_{12}}$. Similarly we can obtain $\frac{\partial a_0^k}{\partial \lambda_{12}^{-}}$.

5. CONCLUSION

The requirements of the weighted aggregation method and the issue of obtaining weights for the weighted aggregation method for fuzzy signatures have been discussed. We also described how to differentiate *min-max* function for gradient based learning of fuzzy signatures. A practical example has been taken to show the technique of calculating derivatives of *max-min* functions for learning.

REFERENCES

[1] T. Vámos, L.T. Kóczy, G. Biró, “Fuzzy Signatures in Data Mining,” *Proc. of the joint 9th IFSA World Congress*, 2001, pp. 2842-2846.

[2] LN Teow, KF Loe, "Effective learning in recurrent max–min neural networks," *Neural Networks*, vol. 11, April 1998, pp. 535-547(13).

[3] L.T. Kóczy, T. Vámos, G. Biró, “Fuzzy Signatures,” *Proc. of EUROFUSE-SIC '99*, 1999, pp. 210-217.

[4] L.T. Kóczy, "Vector Valued Fuzzy Set," Busefal, ete 1980, pp. 41-57.

[5] D. Filev, R. R. Yager, "On the issue of obtaining OWA operator weights," *Fuzzy sets and systems*, vol. 94, 1998, pp. 157-169.

[6] B. S. U. Mendis, T.D. Gedeon, L.T. Kóczy, "Investigation of Aggregation in Fuzzy Signatures,” *3rd International Conference on Computational Intelligence, Robotics and Autonomous Systems, Singapore*, December 2005.